

REVERSE CAUCHY—SCHWARZ INEQUALITIES FOR POSITIVE C^* -VALUED SESQUILINEAR FORMS

MOHAMMAD SAL MOSLEHIAN AND LARS-ERIK PERSSON

Abstract. We prove two new reverse Cauchy–Schwarz inequalities of additive and multiplicative types in a space equipped with a positive sesquilinear form with values in a C^* -algebra. We apply our results to get some norm and integral inequalities. As a consequence, we improve a celebrated reverse Cauchy–Schwarz inequality due to G. Pólya and G. Szegő.

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REFERENCES

- [1] S. S. DRAGOMIR, *A survey on Cauchy–Bunyakovsky–Schwarz type discrete inequalities*, JIPAM. J. Inequal. Pure Appl. Math., **4**, 3 (2003), Article 63, 142 pp.
- [2] S. S. DRAGOMIR, *A counterpart of Schwarz inequality in inner product spaces*, RGMIA. Res. Rep. Coll., **6** (2003), Article 18.
- [3] S. S. DRAGOMIR, *Advances in Inequalities of the Schwarz, Grüss and Bessel Type in Inner Product Spaces*, Nova Science Publishers, New York, 2005.
- [4] S. S. DRAGOMIR, *Advances in Inequalities of the Schwarz, Triangle and Heisenberg Type in Inner Product Spaces*, Nova Science Publishers, Inc., New York, 2007.
- [5] J. I. FUJII, *Operator-valued inner product and operator inequalities*, Banach J. Math. Anal. **2**, 2 (2008), 59–67.
- [6] T. FURUTA, J. M. HOT, J. E. PEČARIĆ AND Y. SEO, *Mond–Pečarić method in operator inequalities. Inequalities for bounded selfadjoint operators on a Hilbert space*, Monographs in Inequalities 1. Zagreb: Element, 2005.
- [7] W. GREUB AND W. RHEINOLDT, *On a generalization of an inequality of L. V. Kantorovich*, Proc. Amer. Math. Soc., **10** (1959), 407–415.
- [8] D. ILISEVIĆ AND S. VAROSANEC, *On the Cauchy–Schwarz inequality and its reverse in semi-inner product C^* -modules*, Banach J. Math. Anal., **1** (2007), 78–84.
- [9] M. JOIȚA, *On the Cauchy–Schwarz inequality in C^* -algebras*, Math. Rep. (Bucur.), **3(53)**, 3 (2001), 243–246.
- [10] E. C. LANCE, *Hilbert C^* -Modules*, London Math. Soc. Lecture Note Series 210, Cambridge Univ. Press, 1995.
- [11] V. M. MANUILOV AND E. V. TROITSKY, *Hilbert C^* -Modules*, Translations of Mathematical Monographs, 226. American Mathematical Society, Providence, RI, 2005.
- [12] G. J. MURPHY, *C^* -algebras and Operator Theory*, Academic Press, Boston, 1990.
- [13] C. P. NICULESCU, *Converses of the Cauchy–Schwarz inequality in the C^* -framework*, An. Univ. Craiova Ser. Mat. Inform., **26** (1999), 22–28.
- [14] N. OZEKI, *On the estimation of the inequalities by the maximum, or minimum values*, J. College Arts Sci. Chiba Univ., **5**, 2 (1969), 199–203.
- [15] G. PÓLYA AND G. SZEGŐ, *Aufgaben und Lehrätze aus der Analysis. Band 1: Reihen, Integralrechnung, Funktionentheorie* (in German), 4th Ed., Springer-Verlag, Berlin, 1970 (original version: Julius Springer, Berlin, 1925).

- [16] G. PÓLYA AND G. SZEGÖ, *Problems and theorems in analysis. Vol. I: Series, integral calculus, theory of functions*. Translated from the German by D. Aeppli: Die Grundlehren der mathematischen Wissenschaften, Band 193. Springer-Verlag, New York-Berlin, 1972.