

## REMARKS ON $t$ -QUASICONVEX FUNCTIONS

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*Abstract.* Given a convex subset  $D$  of a vector space and a constant  $t \in (0, 1)$ , a function  $f : D \rightarrow \mathbb{R}$  is called  $t$ -quasiconvex if, for all  $x, y \in D$ ,

$$f(tx + (1-t)y) \leq \max\{f(x), f(y)\};$$

$f$  is called *strictly  $t$ -quasiconvex* if, for all  $x, y \in D$  such that  $f(x) \neq f(y)$ ,

$$f(tx + (1-t)y) < \max\{f(x), f(y)\}.$$

The following Kuhn-type theorem is proved: If  $f$  is  $t$ -quasiconvex and strictly  $t$ -quasiconvex then it is  $(1/2)$ -quasiconvex and strictly  $(1/2)$ -quasiconvex. It is also shown that lower semi-continuous strictly  $t$ -quasiconvex functions are quasiconvex, which generalizes the well-known Karamardian's theorem.

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