

HALVING CLOSED CURVES IN NORMED PLANES AND RELATED INEQUALITIES

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Abstract. In an arbitrary normed plane we study the relation between the length of a closed curve and the length of its midpoint curve as well as the length of its image under the halving pair transformation. We show that the image curve under the halving pair transformation is convex provided the original curve is convex. We give a sufficient condition for the geometric dilation of a closed convex curve to be larger than a quarter of the perimeter of the unit circle. Moreover, we obtain several inequalities to show the relation between the halving distance and other quantities well known in convex geometry.

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