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PROPERTIES OF THE INTERMEDIATE POINT FROM THE TAYLOR'S THEOREM

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Abstract. If $I \subseteq \mathbb{R}$ is an interval, $a \in I$ and $f : I \to \mathbb{R}$ is $n \geqslant 1$ times differentiable on I, then, in view of Taylor's theorem, there exists a function $\overline{c} : I \to I$ such that, for each $x \in I$,

$$f(x) = \sum_{k=0}^{n-1} \frac{f^{(k)}(a)}{k!}(x-a)^k + \frac{f^{(n)}(\overline{c}(x))}{n!}(x-a)^n .$$

In this paper we study the behaviour of the derivatives $\overline{c}^{(p)}$ and $\overline{\theta}^{(p)}$ of the functions $\overline{c}$ and $\overline{\theta}$, respectively, when x approaches a, where $\overline{\theta} : I \to]0,1[$ is defined by $\overline{\theta}(x) = (c(x) - a)/(x-a)$, if $x \in I \setminus \{a\}$ and $\overline{\theta}(a) = 1/(n+1)$.

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