Mathematical
Inequalities
& Applications
Volume 12, Number 4 (2009), 773–779

HEPTAGONAL TRIANGLE AS THE EXTREME TRIANGLE OF DIXMIER–KAHANE–NICOLAS INEQUALITY

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Abstract. Let T be a triangle in the Euclidean plane. Let $g(T)$ be the orthic triangle of the triangle T, and let $g^{n+1}(T)$ be the orthic triangle of the triangle $g^n(T)$. In [2] it is proved that for $n \to \infty$ the triangle $g^n(T)$ tends to the point L. It has also been shown that $|OL| \leqslant \frac{4}{3}R$ for all triangles T and that $|OL| = \frac{4}{3}R$ if T is a heptagonal triangle, where (O,R) is the circumscribed circle of the triangle T.

In this paper it will be geometrically proved that the equality in Dixmier–Kahane–Nicolas inequality $|OL| \leqslant \frac{4}{3}R$ is valid in the case of a heptagonal triangle. The relationship between the initial heptagonal triangle T and the obtained point L will also be investigated.

Mathematics subject classification (2000): 51M04.
Keywords and phrases: heptagonal triangle, indirect similarity.

REFERENCES

[1] L. BANKOFF, J. GARFUNKEL, *The heptagonal triangle*, Math. Mag., **46** (1973), 7–19.
[2] J. DIXMIER, J.-P. KAHANE, J.-L. NICOLAS, *Un exemple de non-dérivabilité en géométrie du triangle*, Ens. Math., **53**, 2 (2007), 369–428.
[3] R. KOLAR–ŠUPER, Z. KOLAR–BEGOVIĆ, V. VOLENEC, *Two characterizations of the triangle with the angles* $\frac{\pi}{7}, \frac{2\pi}{7}, \frac{4\pi}{7}$, International Journal of Pure and Applied Mathematics, **44** (2008), 335–338.
[4] V. THÉBAULT, *Bibliographie des triangles et des tétraèdres spéciaux*, Bull. Soc. Roy. Sci. Liège, **19** (1950), 574–607.

Paper MIA-12-60