

WEIGHTED INEQUALITIES FOR A CLASS OF MATRIX OPERATORS: THE CASE $p \leq q$

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Abstract. We prove a new discrete Hardy-type inequality $\|Af\|_{q,u} \leq C\|f\|_{p,v}$, where the matrix operator A is defined by $(Af)_i := \sum_{j=1}^i a_{i,j}f_j$, $a_{i,j} \geq 0$. Moreover, we study the problem of compactness of the operator A , and the dual result is stated.

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