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MONOTONICITY AND CONVEXITY OF S–MEANS

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Abstract. For real α, r, s and positive x, y we define S-means by

$$S(\alpha; r, s; x, y) = \frac{E(r, s; x^{\alpha+1}, y^{\alpha+1})}{E(r, s; x^{\alpha}, y^{\alpha})},$$

where E is the Stolarsky mean. S contains Gini, Heronian and many other known means.
In this paper we investigate convexity properties of $S(\alpha)$ and obtain new inequalities between Gini, Heronian and Stolarsky means.
The results lead to new inequalities for generalized Heronian means and reveal new properties of Stolarsky means.

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