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WEIGHT CHARACTERIZATION OF THE TRACE INEQUALITY FOR THE GENERALIZED RIEMANN–LIOUVILLE TRANSFORM IN $L^{p(x)}$ SPACES

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Abstract. Necessary and sufficient conditions on a weight governing the trace inequality for the Riemann-Liouville transform with variable parameter $R_{\alpha(x)}$ in $L^{p(x)}$ spaces are established provided that p and q satisfy the log-Hölder continuity condition. Weighted criteria for the compactness of $R_{\alpha(x)}$ from $L^{p(x)}$ to $L_v^{q(x)}$ are also derived.

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