SOME INEQUALITIES INVOLVING A FRACTAL OPERATOR OF FUNCTIONS ON THE SPHERE

M. A. NAVASCUES

Abstract. This paper undertakes the construction of fractal versions of the classical spherical harmonics. Some inequalities satisfied by the coefficients of the iterated function systems defining the fractal functions provide sufficient conditions for the existence of new Hilbert bases of functions on the sphere. This fact confirms their properties of good approximation. The methodology used implies the definition of an operator mapping standard functions into their fractal analogues. The transformation is linear and bounded and some upper bounds of its norm are also established.

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