

## FARKAS-TYPE RESULTS FOR GENERAL COMPOSED CONVEX OPTIMIZATION PROBLEMS WITH INEQUALITY CONSTRAINTS

XIAN-JUN LONG, NAN-JING HUANG AND DONAL O'REGAN

*Abstract.* In this paper, we consider a general composed convex optimization problem with inequality systems involving a finite number of convex constraints. We establish the strong duality between the primal problem and the Fenchel-Lagrange dual problem by a conjugate duality approach. Moreover, we obtain some new Farkas-type results for this problem by using weak and strong duality theorems. Our results contain some recent results as special cases.

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