

## SPATIAL DECAY AND BLOW-UP FOR SOLUTIONS TO SOME PARABOLIC EQUATIONS IN THE HALF CYLINDER

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*Abstract.* In this paper, the spatial behaviors of a nonlinear and a quasi-linear parabolic equations with nonlinear boundary conditions are studied on a half cylinder. Under suitable conditions, we get a various, but closely related forms of Phragmén-Lindelöf principle, and we have proved the smooth solution either fails to exist globally, or when it does exist globally, it must tend asymptotically to zero with increasing long distance along the cylinder from the base.

*Mathematics subject classification (2010):* 35B50, 35K55, 35K60.

*Keywords and phrases:* Spatial blow-up, Decay estimates, Nonlinear parabolic equation, Quasi-linear parabolic equation, Initial-boundary conditions.

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