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ON SOME EXTREMALITIES IN THE APPROXIMATE INTEGRATION

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Abstract. Some extremalities for quadrature operators are proved for convex functions of higher order. Such results are known in the numerical analysis, however they are often proved under suitable differentiability assumptions. In our considerations we do not use any other assumptions apart from higher order convexity itself. The obtained inequalities refine the inequalities of Hadamard type. They are applied to give error bounds of quadrature operators under the assumptions weaker from the commonly used.

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