Mathematical
Inequalities
& Applications
Volume 13, Number 1 (2010), 197–201

SERIES INVOLVING THE LEAST AND THE GREATEST PRIME FACTOR OF A NATURAL NUMBER

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Abstract. We determine necessary and sufficient conditions satisfied by the real numbers a, b, c that the series $\sum_{n\geqslant 2} n^a p^b(n) P^c(n)$ is convergent. Here $p(n)$ and $P(n)$ are the least and respectively the greatest prime factor of an integer $n \geqslant 2$.

Mathematics subject classification (2010): 11N05.
Keywords and phrases: Least prime factor, greatest prime factor, sums, series, asymptotic behavior.

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Paper MIA-13-15