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FUNCTIONAL CALCULUS WITH OPERATOR–MONOTONE FUNCTIONS

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Abstract. Let $f(t)$ be a non-negative operator-monotone function defined on $[0,\infty)$, and A,B positive definite operators on a Hilbert space. The inequalities $\langle Bx,x\rangle \leqslant f(\langle Ax,x\rangle)$ for every unit vector x do not imply the operator inequality $B \leqslant f(A)$. We prove, however, that when combined with the inequalities $\langle B^{-1}x,x\rangle^{-1} \geqslant f(\langle A^{-1}x,x\rangle^{-1})$, the relation $B = f(A)$ follows.

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