

## SINGULAR INTEGRAL INEQUALITIES AND NATURAL REGULARIZATIONS

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*Abstract.* We shall introduce natural regularizations for singular integrals from the viewpoint of numerical treatments and establish very good error estimates (fundamental inequalities for potentials) for the regularizations.

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