BOUNDEDNESS OF GENERALIZED HARDY OPERATORS ON WEIGHTED AMALGAM SPACES

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Abstract. Let T_φ^- be the operator defined by

$$T_\varphi^- f(x) = \int_{-\infty}^x \varphi(x-y) f(y) dy,$$

where φ is a positive function on $(0, \infty)$ verifying $\varphi(a+b) \approx \varphi(a) + \varphi(b)$.

In this paper, we characterize the pairs (u, v) of positive measurable functions such that T_φ^- maps the weighted amalgam $(L^{\overline{p}}(v), \ell^{\overline{q}})$ in $(L^p(u), \ell^q)$ for all values of $p, q, \overline{p}, \overline{q}$ with $1 < p, q, \overline{p}, \overline{q} < \infty$.

As particular cases, we characterize some higher order Hardy inequalities in weighted amalgams.

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