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A REFINED REVERSE ISOPERIMETRIC INEQUALITY IN THE PLANE

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Abstract. It is proved that if γ is a closed strictly convex curve in the plane with length L and area A, then

$$L^2 \leqslant 4\pi A + 2\pi|\tilde{A}|,$$

with equality holding if and only if γ is a circle, where $\tilde{A}$ denotes the oriented area enclosed by the locus of curvature centers of γ.

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