

## SOME REMARKS ON CESÀRO-ORLICZ SEQUENCE SPACES

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*Abstract.* In this paper Cesàro-Orlicz spaces, theory of which started in the papers [7], [28] and [10], are investigated. The problem of the necessity of condition  $\delta_2$  for some fundamental topological and geometrical properties is considered again. Criteria for the Kadec-Klee property with respect to the coordinatewise convergence as well as for local uniform convexity of the spaces are given. In the last part, finite dimensional subspaces of Cesàro-Orlicz spaces are investigated.

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