

THE DARBOUX PROBLEM FOR HYPERBOLIC FUNCTIONAL DIFFERENTIAL EQUATIONS AND INEQUALITIES IN THE SENSE OF CARATHÉODORY

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Abstract. We consider the linear and nonlinear problem for partial functional differential equations

$$\frac{\partial^2 u}{\partial x \partial y}(x, y) = C(x, y)u(x, y) + P(x, y)u_{(x, y)} \text{ a.e. in } [0, a] \times [0, b]$$

$$\frac{\partial^2 u}{\partial x \partial y}(x, y) = f(x, y, u_{(x, y)}, u(x, y)) \text{ a.e. in } [0, a] \times [0, b]$$

with Darboux condition

$$u(x, y) = \psi(x, y) \text{ on } [-a_0, a] \times [-b_0, b] \setminus (0, a] \times (0, b]$$

where the Hale operator $u_{(x, y)} : [-a_0, 0] \times [-b_0, 0] \rightarrow \mathbb{R}^n$ is defined by $u_{(x, y)}(s, t) = u(s + x, t + y)$ for $(s, t) \in [-a_0, 0] \times [-b_0, 0]$. We give a few theorems about weak and strong inequalities and the existence theorem for the nonlinear problem.

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