

## NEW BOUNDS FOR SOLUTIONS OF A SINGULAR INTEGRO–DIFFERENTIAL INEQUALITY

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*Abstract.* New bounds for solutions of an integro-differential inequality with weakly singular kernel are established using a weighted version of the Hardy-Littlewood-Sobolev inequality. Besides, some applications to real world problems are presented.

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