MINIMAX INEQUALITY FOR THE PREDICTION RISK

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Abstract. This is a continuation of the article "Sharp inequality for the Bayes prediction risk" (2007). But, instead of the Bayes prediction problem, the minimax one is considered in this paper. The lower bound for the minimax prediction risk is derived by using the information inequality. Moreover it is shown that a predictor $\delta(X)$ for which the lower bound is attained is a minimax one. Sufficient conditions for the admissibility of predictors are also given.

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