KERNEL OPERATORS WITH VARIABLE INTERVALS OF INTEGRATION IN LEBESGUE SPACES AND APPLICATIONS

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Abstract. New criteria of $L_p - L_q$ boundedness of Hardy-Steklov type operator (**??**) with both increasing on $(0,\infty)$ boundary functions $a(x)$ and $b(x)$ are obtained for $1 < p \leqslant q < \infty$ and $0 < q < p < \infty$, $p > 1$. This result is applied for two-weighted $L_p - L_q$ characterization of the corresponding geometric Steklov operator (**??**) and other related problems.

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