

INTEGRAL INEQUALITIES AND THEIR APPLICATIONS TO THE CALCULUS OF VARIATIONS ON TIME SCALES

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Abstract. We discuss the use of inequalities to obtain the solution of certain variational problems on time scales.

Mathematics subject classification (2010): 26D15, 49K05, 39A10.

Keywords and phrases: Time scales, calculus of variations, optimal control, integral inequalities, Jensen's inequality, global minimizers.

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