FURTHER DEVELOPMENTS OF FURUTA
INEQUALITY OF INDEFINITE TYPE

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Abstract. A selfadjoint involutive matrix $J$ endows $\mathbb{C}^n$ with an indefinite inner product $[\cdot, \cdot]$ given by $[x, y] := \langle Jx, y \rangle$, $x, y \in \mathbb{C}^n$. We study matrix inequalities for $J$-selfadjoint matrices with nonnegative eigenvalues. Namely, Furuta inequality of indefinite type is revisited. Characterizations of the $J$-chaotic order and of the $J$-relative entropy are obtained via Furuta inequality. The parallelism between the definite versions of the inequalities on Hilbert spaces and the corresponding indefinite versions on Krein spaces is pointed out.


Keywords and phrases: Krein space, $J$-selfadjoint matrix, Furuta inequality, $J$-chaotic order, $J$-relative entropy.

REFERENCES

[4] T. FURUTA, $A \succeq B \succeq 0$ assures $(B^p A^q B^r)^{1/q} \succeq B^{(p+2r)/q}$ for $r \geq 0$, $p \geq 0$, $q \geq 1$ with $(1+2r)q \geq p + 2r$, Proc. Amer. Math. Soc. 101 (1987), 85–88.