FURTHER DEVELOPMENTS OF FURUTA
INEQUALITY OF INDEFINITE TYPE

N. BEBIANO, R. LEMOS, J. DA PROVIDÊNCIA AND G. SOARES

Abstract. A selfadjoint involutive matrix J endows $\mathbb{C}^n$ with an indefinite inner product $[\cdot,\cdot]$ given by $[x,y] := \langle Jx, y \rangle$, $x, y \in \mathbb{C}^n$. We study matrix inequalities for J-selfadjoint matrices with nonnegative eigenvalues. Namely, Furuta inequality of indefinite type is revisited. Characterizations of the J-chaotic order and of the J-relative entropy are obtained via Furuta inequality. The parallelism between the definite versions of the inequalities on Hilbert spaces and the corresponding indefinite versions on Krein spaces is pointed out.

Mathematics subject classification (2010): 47B50, 47A63, 15A45.
Keywords and phrases: Krein space, J-selfadjoint matrix, Furuta inequality, J-chaotic order, J-relative entropy.

REFERENCES

[1] T. YA. AZIZOV, I. S. IOKHVIDOV, *Linear Operators in Spaces with an Indefinite Metric*, Nauka, Moscow, 1986, English Translation: Wiley, New York, 1989.
[2] T. ANDO, *Löwner inequality of indefinite type*, Linear Algebra Appl. **385** (2004), 73–80.
[3] J. I. FUJII AND E. KAMEI, *Relative operator entropy in noncommutative information theory*, Math. Japonica **3** (1989), 341–348.
[4] T. FURUTA, $A \geqslant B \geqslant 0$ *assures* $(B^r A^p B^r)^{1/q} \geqslant B^{(p+2r)/q}$ *for* $r \geqslant 0$, $p \geqslant 0$, $q \geqslant 1$ *with* $(1+2r)q \geqslant p+2r$, Proc. Amer. Math. Soc. **101** (1987), 85–88.
[5] T. FURUTA, *Furuta's inequality and its application to the relative operator entropy*, J. Oper. Theory **30** (1993), 21–30.
[6] T. FURUTA, *Characterizations of chaotic order via generalized Furuta inequality*, J. Inequal. Appl. **1** (1) (1997), 11–24.
[7] T. FURUTA, *Invitation to Linear Operators*, Taylor & Francis, London, 2001.
[8] T. SANO, *Furuta inequality of indefinite type*, Math. Inequal. Appl. **10** (2007), 381–387.
[9] T. SANO, *On chaotic order of indefinite type*, J. Inequal. Pure Appl. Math. **8** (3) (2007), Art. 62, 4 p.

Mathematical Inequalities & Applications
www.ele-math.com
mia@ele-math.com