HARDY INEQUALITIES WITH REMAINDER TERMS FOR THE GENERALIZED BAOUENDI–GRUSHIN VECTOR FIELDS

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Abstract. Based on the properties of vector fields and the generalized divergence formula, we prove the Hardy inequalities with remainder terms for the generalized Baouendi-Grushin vector fields and determine the best constants in these Hardy inequalities.
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