

## FUNCTIONAL CHARACTERIZATION OF A SHARPENING OF THE TRIANGLE INEQUALITY

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*Abstract.* Motivated by recent refinements of the classical triangle inequality in normed spaces proved by Lech Maligranda we deal with related functional equation and inequality and with the corresponding stability problem.

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