SCHUR'S INEQUALITY FOR THE DIMENSION
OF COMMUTING FAMILIES OF MATRICES

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Abstract. We present an elementary proof for the above inequality. This in particular leads us easily to see that the maximal linearly independent sets of commuting $n \times n$ matrices, for $n \geqslant 4$, necessarily consist of either nilpotent elements or units.

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