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SCHUR CONVEXITY AND HADAMARD'S INEQUALITY

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Abstract. Suppose that I is an open interval and $f : I \to \mathbb{R}$ is a continuous function. If

$$F(x,y) = \begin{cases} \frac{1}{y-x}\int_x^y f(t)dt - f(\frac{x+y}{2}), & x,y \in I, x \neq y, \\ 0, & x = y \in I, \end{cases}$$

and

$$G(x,y) = \begin{cases} \frac{f(x)+f(y)}{2} - \frac{1}{y-x}\int_x^y f(t)dt, & x,y \in I, x \neq y, \\ 0, & x = y \in I, \end{cases}$$

then $F(x,y)$ and $G(x,y)$ are Schur convex (concave) on I^2 if and only if f is convex (concave) on I.

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