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REFINED GENERALIZATIONS OF THE TRIANGLE INEQUALITY ON BANACH SPACES

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Abstract. Let X be a complex Banach space and p a real number with $p \geqslant 1$. We give a necessary and sufficient condition for complex numbers a, b and real numbers λ, μ and ν in order that the inequality

$$\frac{\|ax+by\|^p}{\lambda} \leqslant \frac{\|x\|^p}{\mu} + \frac{\|y\|^p}{\nu}$$

holds for every $x, y \in X$.

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