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SHARP MEAN TRIANGLE INEQUALITY

MASATOSHI FUJII, MIKIO KATO, KICHI-SUKE SAITO
AND TAKAYUKI TAMURA

Abstract. By using a mean operator we shall present some sharp mean triangle inequalities in a Banach space which generalize the sharp triangle inequality with n elements and its reverse one shown recently by the last three authors in [7]. In the course of doing this we shall present a new two element triangle inequality with parameter and its reverse. Several applications will be given.

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