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A REMARK ABOUT POSITIVE POLYNOMIALS

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Abstract. The following theorem is proved.

THEOREM. *Let* $P(x)=\sum_{k=0}^{2n}a_kx^k$ *be a polynomial with positive coefficients. If the inequalities* $\frac{a_{2k+1}^2}{a_{2k}a_{2k+2}}<\frac{1}{\cos^2(\frac{\pi}{n+2})}$ *hold for all* $k=0,1,\dots,n-1$, *then* $P(x)>0$ *for every* $x\in\mathbb{R}$.

We show that the constant $\frac{1}{\cos^2(\frac{\pi}{n+2})}$ in this theorem cannot be increased. We also present some corollaries of this theorem.

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REFERENCES

[1] F. R. GANTMAHER, *Matrix Theory*, 4-th. ed., Nauka, Moscow, 1988 (Russian). English translation in: F. R. Gantmaher, *The Theory of Matrices*, New York: Chelsea Pub. Co., 1959.
[2] G. H. HARDY, *On the zeros of a class of integral functions*, Messenger of Math., **34** (1904), 97–101.
[3] G. H. HARDY, *Collected Papers of G. H. Hardy,* vol. IV, Oxford Clarendon Press, 1969.
[4] J. I. HUTCHINSON, *On a remarkable class of entire functions*, Trans. Amer. Math. Soc., **25** (1923), 325–332.
[5] O.M.KATKOVA, A.M.VISHNYAKOVA, *On sufficient conditions for the total positivity and for the multiple positivity of matrices*, Linear Algebra Appl., **416** (2006), 1083–1097.
[6] O.M.KATKOVA, A.M.VISHNYAKOVA, *A sufficient condition for a polynomial to be stable*, Journal of Mathematical Analysis and Applications, **347** (2008), 81–89.
[7] D. C. KURTZ, *A suffcient condition for all the roots of a polynomial to be real*, Amer. Math. Monthly, **99** (1992), 259–263.
[8] PETROVITCH, *Une classe remarquable de séries entiéres*, Atti del IV Congresso Internationale dei Matematici, Rome,(Ser. 1), **2** (1908), 36–43.

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