CLASSIFIED CONSTRUCTION OF GENERALIZED
FURUTA TYPE OPERATOR FUNCTIONS, II

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Abstract. As a continuation of our previous work with the same title, the construction of grand Furuta inequality (GFI) is improved in order to give a constructive proof of Uchiyama's result (2003). Afterwards the generalization of Uchiyama's result is obtained by mathematical induction which implies Furuta's recent results.

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