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ON GENERALIZATION OF MOSER'S THEOREM IN THE CRITICAL CASE

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Abstract. Let Ω be an open bounded set in $\mathbb{R}^n$, $n \geqslant 2$. In paper [13] Moser proved that for every $K \geqslant K_0 = n^{-\frac{n-1}{n}}\,\omega_{n-1}^{-\frac{1}{n}}$ we have

$$\sup\Big\{\int_\Omega \exp\Big(\Big(\frac{f(x)}{K}\Big)^{\frac{n}{n-1}}\Big) : f \in W_0^{1,n}(\Omega), ||\nabla f||_{L^n} \leqslant 1\Big\} < \infty\,,$$

but for $K < K_0$ the supremum is not finite.

In this paper we study the critical case $K = K_0$ for arbitrary Orlicz-Sobolev spaces with Young functions that behave like t^n close to ∞. We show that for functions like $t^n(1-\log^{-a} t)$ the supremum is finite for $a > 1$ but infinite for $0 < a < 1$.

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