

## A NEW METHOD FOR ESTABLISHING AND PROVING ACCURATE BOUNDS FOR THE WALLIS RATIO

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*Abstract.* The aim of this paper is to establish new inequalities about the Wallis ratio that improve the Gautschi-Kershaw results.

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