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HILBERT–TYPE INEQUALITIES AND RELATED OPERATORS WITH HOMOGENEOUS KERNEL OF DEGREE 0

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Abstract. In this paper we provide an unified approach to the Hilbert-type inequalities with homogeneous kernel of degree 0 and certain weight functions. As an application, we define the related Hilbert-type operators and analyze their norms. In the case of conjugate exponents, we obtain the best possible constants involved in the right-hand sides of derived inequalities, and norms of the Hilbert-type operators as well. Finally, we consider some special choices of homogeneous kernels and parameters.

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REFERENCES

[1] A. BÉNYI AND C. D. OH, *Best constants for certain multilinear integral operators*, Journ. Ineq. Appl, Article ID 28582 (2006), 1–12.
[2] Y. BICHENG, *On extension of Hilbert's integral inequality with some parameters*, The Austral. Journ. of Math. Anal. and Appl., **1**, 1 (2004), 1–8.
[3] Y. BICHENG, I. BRNETIĆ, M. KRNIĆ, AND J. PEČARIĆ, *Generalization of Hilbert and Hardy-Hilbert integral inequalities*, Math. Inequal. Appl., **8**, 2 (2005), 259–272.
[4] Y. BICHENG, *On the norm of an integral operator and applications*, Journ. Math. Anal. Appl., **321** (2006), 182–192.
[5] Y. BICHENG, *A Hilbert-type integral inequality*, Journ. Zheijang Univ. (Science Edition), **24**, 2 (2007), 121–124.
[6] Y. BICHENG, *A relation to Hilbert's integral inequality and some basic Hilbert-type inequalities*, Journ. of Ineq. in Pure and Appl. Math, **9**, 2 (2008), 1–8.
[7] Y. BICHENG, *The norm of operator and Hilbert-type inequalities*, Science Press, Beijing, 2009.
[8] F. F. BONSALL, *Inequalities with non-conjugate parameters*, Quart. J. Math. Oxford Ser., **2**, 2 (1951), 135–150.
[9] A. ČIŽMEŠIJA, M. KRNIĆ, AND J. PEČARIĆ, *General Hilbert-type inequalities with non-conjugate exponents*, Math. Inequal. Appl., **11**, 2 (2008), 237–269.
[10] G. H. HARDY, *Note on a theorem of Hilbert concerning series of positive term*, Proc. of London Math. Soc., **23**, 2 (1925), Records of Proc. xlv–xlvi.
[11] G. H. HARDY, J. E. LITTLEWOOD, AND G. PÓLYA, *Inequalities*, 2nd edition, Cambridge University Press, Cambridge, 1967.
[12] M. KRNIĆ AND J. PEČARIĆ, *General Hilbert's and Hardy's inequalities*, Math. Inequal. Appl., **8**, 1 (2005), 29–51.
[13] M. KRNIĆ, G. MINGZHE, J. PEČARIĆ, AND G. XUEMEI, *On the best constant in Hilbert's inequality*, Math. Inequal. Appl., **8**, 2 (2005), 317–329.
[14] M. KRNIĆ, J. PEČARIĆ, AND P. VUKOVIĆ, *On some higher-dimensional Hilbert's and Hardy-Hilbert's integral inequalities with parameters*, Math. Inequal. Appl., **11**, 4 (2008), 701–716.
[15] M. KRNIĆ, *Hilbert inequality and Gaussian hypergeometric functions*, Journ. Math. Ineq. (to appear).
[16] J. KUANG, *Introduction to real analysis*, Hunan Education Press, Chansha, 1996.

Paper MIA-13-59

[17] V. LEVIN, *On the two-parameter extension and analogue of Hilbert's inequality*, J. London Math. Soc., **11** (1936), 119–124.
[18] D. S. MITRINOVIĆ, J. E. PEČARIĆ, AND A. M. FINK, *Classical and new inequalities in analysis*, Kluwer Academic Publishers, Dordrecht/Boston/London, 1993.