BOUNDEDNESS OF GENERALIZED RIESZ
POTENTIALS ON SPACES OF HOMOGENEOUS TYPE

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Abstract. The authors introduce a class of generalized Riesz potentials with kernels having weak regularity on spaces of homogeneous type in the sense of Coifman and Weiss and establish their boundedness on Lebesgue spaces and Hardy spaces. As applications, the authors obtain the boundedness on Lebesgue spaces and Hardy spaces of commutators generated by Lipschitz functions and generalized Riesz potentials or Calderón-Zygmund operators with kernels having weak regularity on spaces of homogeneous type.

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