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A NEW GENERALIZATION OF HARDY–HILBERT'S INEQUALITY WITH NON–HOMOGENEOUS KERNEL

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Abstract. Let $p > 1$, $1/p + 1/p^* = 1$, and $a = (a_n)_{n=1}^{\infty}$, $b = (b_m)_{m=1}^{\infty}$ be two complex sequences. We exhibit the generalization of Hardy-Hilbert's inequality of the following type:

$$\sum_{n,m\geqslant 1} K(\phi_1(n),\phi_2(m))|a_n||b_m| < C\left(\sum_{n=1}^{\infty}|\frac{a_n}{f_1(\phi_1(n))}|^p\right)^{\frac{1}{p}}\left(\sum_{m=1}^{\infty}|\frac{b_m}{f_2(\phi_2(m))}|^{p^*}\right)^{\frac{1}{p^*}},$$

where $K : (0,\infty)\times(0,\infty)\to(0,\infty)$, $f_1, f_2, \phi_1, \phi_2 : (0,\infty)\to(0,\infty)$ and C is a suitable constant. We also get several interesting inequalities which generalize many recent results.

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REFERENCES

[1] L. E. AZAR, *On some extensions of Hardy-Hilbert's inequality and applications*, J. Ineq. Appl., vol. **2008**, Article ID 546829, 14 pages, 2008.
[2] I. BRNETIĆ, J. PEČARIĆ, *Generalization of inequalities of Hard-Hilbert type*, Math. Ineq. Appl., **7** (2004) 217–225.
[3] G. H. HARDY, J. E. LITTLEWOOD, G. PÓLYA, *Inequalities*, 2nd edition, Cambridge University Press, Cambridge, 1967.
[4] M. KRNIĆ, J. PEČARIĆ, *Extension of Hilbert's inequality*, J. Math. Anal. Appl., **324** (2006), 150–160.
[5] W. RUDIN, *Principles of mathematical analysis*, 3rd edition, McGraw-Hill, Inc., New York, 1976.
[6] W. RUDIN, *Real and complex analysis*, 3rd edition, Academic Press, New York, 1966.
[7] B. YANG, *On a relation between Hilbert's inequality and a Hilbert-type inequality*, Appl. Math. Lett., **21** (2008), 483–488.
[8] B. YANG, L. DEBNATH, *On the extended Hardy-Hilbert's inequality*, J. Math. Anal. Appl., **272** (2002), 187–199.

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