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A CHARACTERIZATION OF THE SINE FUNCTION BY FUNCTIONAL INEQUALITIES

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Abstract. In the present paper we deal with the first generalization of Wilson's difference

$$f(x)g(y)-f\left(\frac{x+y}{2}\right)^2+f\left(\frac{x-y}{2}\right)^2,$$

assuming that its absolute value is majorized by some function in a single variable.

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