THE HARNACK INEQUALITY FOR THE RIEMANN–LIOUVILLE
FRACTIONAL DERIVATION OPERATOR

Rico Zacher

Abstract. In this note we establish the Harnack inequality for the Riemann-Liouville fractional derivation operator ∂_t^α of order $\alpha \in (0,1)$. Here the function under consideration is assumed to be globally nonnegative. We show that the Harnack inequality in general fails if this global positivity assumption is replaced by a local one. A Harnack estimate is also derived for nonnegative solutions of a class of nonhomogeneous fractional differential equations.

Mathematics subject classification (2010): 26A33, 45D05, 47G20.
Keywords and phrases: Harnack inequality, fractional derivative, fractional differential equation.

REFERENCES

[1] R. F. BASS, M. KASSMANN, *Harnack inequalities for non-local operators of variable order*, Trans. Amer. Math. Soc. **357**, 2 (2005), 837–850.

[2] L. CAFFARELLI, L. SILVESTRE, *An extension problem related to the fractional Laplacian*, Comm. Partial Differential Equations, **32** (2007), 1245–1260.

[3] E. DIBENEDETTO, *Degenerate parabolic equations*, Universitext, Springer-Verlag, New York, 1993.

[4] E. DIBENEDETTO, *Partial differential equations*, Birkhäuser, 1995.

[5] D. GILBARG, N. TRUDINGER, *Elliptic partial differential equations of second order*, Springer, 1977.

[6] J. HADAMARD, *Extension à l'équation de la chaleur d'un théorème de A. Harnack*, Rend. Circ. Mat. Palermo, **3** (1954), 337–346.

[7] M. KASSMANN, *Harnack inequalities: an introduction*, Bound. Value Probl. 2007, Art. ID 81415, 21 pp.

[8] M. KASSMANN, *The classical Harnack inequality fails for non-local operators*, preprint.

[9] A. A. KILBAS, H. M. SRIVASTAVA, J. J. TRUJILLO, *Theory and applications of fractional differential equations*, Elsevier, 2006.

[10] G. M. LIEBERMAN, *Second order parabolic differential equations*, World Scientific, 1996.

[11] J. MOSER, *A Harnack inequality for parabolic differential equations*, Comm. Pure Appl. Math., **17** (1964), 101–134. Correction in Comm. Pure Appl. Math., **20** (1967), 231–236.

[12] B. PINI, *Sulla soluzione generalizzata di Wiener per il primo problema di valori al contorno nel caso parabolico*, Rend. Sem. Mat. Univ. Padova, **23** (1954), 422–434.

[13] S. G. SAMKO, A. A. KILBAS, O. I. MARICHEV, *Fractional integrals and derivatives. Theory and applications*, Gordon and Breach Science Publishers, Yverdon, 1993.

[14] R. ZACHER, *A weak Harnack inequality for fractional differential equations*, J. Integral Equations Appl., **19** (2007), 209–232.

[15] R. ZACHER, *Boundedness of weak solutions to evolutionary partial integro-differential equations with discontinuous coefficients*, J. Math. Anal. Appl., **348** (2008), 137–149.

© ELEMENT, Zagreb
Paper MIA-14-03

Mathematical Inequalities & Applications
www.ele-math.com
mia@ele-math.com