

AN ALTERNATIVE PROOF OF ELEZOVIĆ–GIORDANO–PEČARIĆ’S THEOREM

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Abstract. In this paper, an alternative proof is supplied for monotonicity and convexity of the function $z_{s,t}(x) = [\Gamma(x+t)/\Gamma(x+s)]^{1/(t-s)} - x$ with $z_{s,s}(x) = e^{\psi(x+s)} - x$, where Γ is the classical Euler’s gamma function, s and t are real numbers with $t - s \neq \pm 1$, $\alpha = \min\{s, t\}$ and $x \in (-\alpha, \infty)$.

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