

SOME GENERALIZATIONS FOR OPIAL'S INEQUALITY INVOLVING SEVERAL FUNCTIONS AND THEIR DERIVATIVES OF ARBITRARY ORDER ON ARBITRARY TIME SCALES

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Abstract. In this paper, some various types of Opial's inequality involving several functions and their higher-order derivatives are presented on arbitrary time scales. The well-known Muirhead's inequality is employed to obtain very interesting results. While dealing with higher-order derivatives, the generalized Taylor's formula and the generalized polynomials are used to simplify our proofs too. Our new results generalize and extend the existing results in the literature, and some of the works done by Pachpatte. Moreover, our results are not only new for arbitrary time scales, but also new for the continuous and the discrete cases.

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