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BOUNDING EXPECTATIONS OF FUNCTIONS OF RANDOM VECTORS WITH GIVEN MARGINALS AND SOME MOMENTS: APPLICATIONS OF THE MULTIVARIATE DISCRETE MOMENT PROBLEM

GERGELY MÁDI-NAGY AND ANDRÁS PRÉKOPA

Abstract. The paper shows how the bounding technique provided by the multivariate discrete moment problem can be used for bounding expectations of functions of random variables with known univariate marginals and some of the mixed moments. Four examples are presented. In the first one the function is a Monge or related type array, in the second one it is a pseudo-Boolean function. In the further examples bounds are given for values of multivariate generating functions and expectations of special utility functions of random vectors. Numerical results are presented.

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REFERENCES

[1] K. ARROW, *Essays in the Theory of Risk Bearing* Chap. 3, North-Holland, Amsterdam, 1970.
[2] BEIN, W. W., P. BRUCKER, J. K. PARK AND P. K. PATHAK, *A Monge Property for the d-dimensional Transportation Problem*, Discrete Applied Mathematics, **58** (1990), 97–109.
[3] J. BUKSZÁR, *Hypermultitrees and sharp Bonferroni inequalities*, Math. Inequal. Appl., **6**, 4 (2003), 727–743.
[4] BUKSZÁR, J. AND A. PRÉKOPA, *Probability Bounds with Cherry-Trees*, Mathematics of Operations Research, **26** (2001), 174–192.
[5] BUKSZÁR, J. AND T. SZÁNTAI, *Probability bounds given by hypercherry trees*, Optimization Methods and Software, **17** (2002), 409–422.
[6] J.CABALLE, AND A. POMANSKY, *Mixed Risk Aversion*, Journal of Economic Theory, **71** (1996), 485–513.
[7] A.J. HOFFMAN, *On Simple Linear Programming Problems*, V. Klee,ed, Proceedings of Symposia in Pure Mathematics, Convexity **Vol. VII** (1963), (AMS, Providence, RI), 317–327.
[8] HOU, X. AND A. PRÉKOPA, *Monge Property and Bounding Multivariate Probability Distribution Functions with Given Marginals and Covariances*, SIAM Journal on Optimiziation, **18**, 1 (2007), 138–155.
[9] A. IBRAHIM, A. R. MUGDADI, *Bounds of moment generating functions of some life distributions*, Statist. Papers, **46**, 4 (2005), 575–585.
[10] R.L. KEENEY AND H. RAIFFA, *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*, Wiley, New York, 1976.
[11] KUAI, H., F. ALAJAJI AND G. TAKAHARA, *A Lower Bound on the Probability of a Finite Union of Events*, Discrete Mathematics, **215**, 2 (2000), 147–158.
[12] G. MÁDI-NAGY, *A method to find the best bounds in a multivariate discrete moment problem if the basis structure is given*, Studia Scientiarum Mathematicarum Hungarica, **42**, 2 (2005), 207–226.
[13] G. MÁDI-NAGY, *On Multivariate Discrete Moment Problems: Generalization of the Bivariate Min Algorithm for Higher Dimensions*, SIAM Journal on Optimization, **19**, 4 (2009), 1781–1806.

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[14] G. MÁDI-NAGY, AND A. PRÉKOPA, *On Multivariate Discrete Moment Problems and Their Applications to Bounding Expectations and Probabilities*, Mathematics of Operations Research, **29**, 2 (2004), 229–258.
[15] T. POPOVICIU, *Les Fonctions Convexes*, Actualités Scientifiques et Industrielles 992, Hermann, Paris, 1944.
[16] A. PRÉKOPA, *The Discrete Moment Problem and Linear Programming*, Discrete Applied Mathematics, **27** (1990), 235–254.
[17] A. PRÉKOPA, *Bounds on Probabilities and Expectations Using Multivariate Moments of Discrete Distributions*, Studia Scientiarum Mathematicarum Hungarica, **34** (1998), 349–378.
[18] A. PRÉKOPA, AND L. GAO, *Bounding the Probability of the Union of Events by Aggregation and Disaggregation in Linear Programs*, Discrete Applied Mathematics, **145** (2005), 444–454.
[19] A. PRÉKOPA, AND G. MÁDI-NAGY, *A Class of Multiattribute Utility Functions*, Economic Theory, **34**, 3 (2008), 591–602.
[20] A. PRÉKOPA, B. VIZVÁRI, G. REGŐS, *Lower and Upper Bounds on Probabilities of Boolean Function of Events*, RUTCOR Research Report, 1997, 21–97.
[21] S.M. ROSS, *Introduction to Probability Models*, Wiley, New York, 2002.
[22] B. VIZVÁRI, *New upper bounds on the probability of events based on graph structures*, Math. Inequal. Appl., **10**, 1 (2007), 217–228.