L^p –ANALOGUES OF BERNSTEIN AND MARKOV INEQUALITIES

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Abstract. Let $\|\cdot\|_\infty$ denote the sup norm on $[-1,1]$. If $x \in [-1,1]$ is fixed and $\mathscr{M}_{m,n}(x)$ is the best constant in

$$|p'(x)| \leqslant \mathscr{M}_{m,n}(x)\|p\|_\infty,$$

for all trinomials p of the form $p(x) = ax^m + bx^n + c$ with $a,b,c \in \mathbb{R}$, then the exact value of $\mathscr{M}_{m,n}(x)$ is known for large families of pairs $(m,n) \in \mathbb{N}^2$. Here we consider the same problem for L^p-norms.

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