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SINGULAR INTEGRALS WITH MIXED HOMOGENEITY IN PRODUCT SPACES

LARRY CHEN AND HUNG VIET LE

Abstract. Let $\Omega \in L(\log L^+)^2(S^{n-1}\times S^{m-1})$ $(n,m\geqslant 2)$ satisfy some cancellation conditions. We prove the L^p boundedness $(1<p<\infty)$ of the singular integral

$$Tf(x_1,x_2)=\text{p. v.}\int\!\!\int_{\mathbb{R}^n\times\mathbb{R}^m}\frac{\Omega(y_1',y_2')h(\rho_1(y_1),\rho_2(y_2))}{\rho_1^{\alpha}(y_1)\rho_2^{\beta}(y_2)}f(x_1-y_1,x_2-y_2)\,dy_1\,dy_2,$$

where ρ_1, ρ_2 are some metrics which are homogeneous with respect to certain non-isotropic dilations. We also study the above singular integral along some surfaces.

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