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THE BEST CONSTANT FOR THE CENTERED MAXIMAL OPERATOR ON RADIAL DECREASING FUNCTIONS

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Abstract. We show that the lowest constant appearing in the weak type (1,1) inequality satisfied by the centered Hardy-Littlewood maximal operator on radial, radially decreasing integrable functions is 1.

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