

ON THE ANALOG OF SHEPHARD PROBLEM FOR THE L_p -PROJECTION BODY

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Abstract. For $p \geq 1$, Lutwak, Yang and Zhang introduced the concept of L_p -projection body. In this paper, we develop a Fourier analytic approach in the L_p -Brunn-Minkowski theory. We consider the question of whether $\Pi_p K \subseteq \Pi_p L$ implies $\Omega_p(K) \leq \Omega_p(L)$. We also formulate and solve an analog of the Shephard problem for the L_p -projection body.

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