

## STIRLING'S FORMULA REVISITED VIA SOME CLASSICAL AND NEW INEQUALITIES

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*Abstract.* The Hermite-Hadamard inequality is used to develop an approximation to the logarithm of the gamma function which is more accurate than the Stirling approximation and easier to derive. Then the concavity of the logarithm of gamma of logarithm is proved and applied to the Jensen inequality. Finally, the Wallis ratio is used to obtain the additional term in Stirling's approximation formula.

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