BRÉZIS–GALLOUËT–WAINGER INEQUALITY
WITH A DOUBLE LOGARITHMIC TERM ON A
BOUNDED DOMAIN AND ITS SHARP CONSTANTS

KEI MORII, TOKUSHI SATO AND HIDEMITSU WADADE

Abstract. The Brézis-Gallouët-Wainger inequality gives an estimate of the $L^\infty$-norm by the critical Sobolev norm with the aid of the logarithmic dependency of a higher order Sobolev norm. We investigate the Brézis-Gallouët-Wainger inequality on a bounded domain with the first or- der critical Sobolev space, and give the best constant in the inequality in some special cases. Furthermore, since the inequality does not hold with the sharp constant, we add a double logarithmic term and give the sharp constant for its coefficient. A part of our results is mainly based on an investigation of the inequality with the higher-order Sobolev norm replaced by the Hölder seminorm.


Keywords and phrases: Sobolev embedding theorem, Brézis-Gallouët-Wainger inequality, sharp constants, double logarithmic term.

REFERENCES