

## ON WEIGHTED MEAN MATRICES WHOSE $l^p$ NORMS ARE DETERMINED ON DECREASING SEQUENCES

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*Abstract.* We give a condition on weighted mean matrices so that their  $l^p$  norms are determined on decreasing sequences when the condition is satisfied. We apply our result to give a proof of a conjecture of Bennett and discuss some related results.

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