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ON A WEIGHTED SUM OF DISTANCES FROM A WELL DISTRIBUTED SET OF POINTS

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Abstract. Suppose that ξ is a complex number, $t>0$ and $w_1,\dots,w_d \geqslant 0$. Let Δ be the modulus of the product of $d(d-1)/2$ distances between complex numbers $z_1,\dots,z_d$ labelled so that $|z_1-\xi| \geqslant \dots \geqslant |z_d-\xi|$. We prove that the sum $\frac{1}{d}\sum_{i=1}^{d} w_i|z_i-\xi|^t$ is at least

$$\frac{\sqrt{e}}{2}e^{-1/d}\Delta^{2t/d(d-1)}d^{-t/(d-1)}\prod_{i=1}^{d-1} w_i^{2(d-i)/d(d-1)}$$

and show that this inequality is sharp for certain choice of weights w_i. This inequality is then applied to sets of conjugate algebraic integers.

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